

Spacetime Foam^{*}

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Abstract

Spacetime is composed of a fluctuating arrangement of bubbles or loops called spacetime foam, or quantum foam. We use the holographic principle to deduce its structure, and show that the result is consistent with gedanken experiments involving spacetime measurements. We propose to use laser-based atom interferometry techniques to look for spacetime fluctuations. Our analysis makes it clear that the physics of quantum foam is inextricably linked to that of black holes. A negative experimental result, therefore, might have non-trivial ramifications for semiclassical gravity and black hole physics.

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Spacetime appears smooth on large scales. On small scales, however, it is bubbly and foamy due to quantum fluctuations. In this essay, we use the holographic principle to show that the fluctuations are much larger than what conventional wisdom leads us to believe. We alternatively derive the same results by carrying out gedanken experiments to measure distances and time intervals. Intriguingly, the fluctuations are large enough that they may one day be detectable with improved modern laser-based atom interferometers.

From the holographic principle to spacetime foam

The holographic principle grew out of the profound insights of Wheeler, Bekenstein, Hawking, 't Hooft, and Susskind.[1] It states that the maximum number of degrees of freedom that can be put into a region of space is given by the area of the region in Planck units. To connect it to quantum foam, let us consider a region of space measuring $R \times R \times R$, and imagine partitioning it into cubes as small as physical laws allow. With each small cube we associate one degree of freedom. If the smallest uncertainty in measuring a distance R is δR , in other words, if the fluctuation in distance R is δR , then the smallest such cubes have volume $(\delta R)^3$. (Otherwise, one could divide R into units each measuring less than δR , and by counting the number of such units in R , one would be able to measure R to within an uncertainty smaller than δR .) Thus the maximum number of degrees of freedom, given by the number of small cubes we can put into the region of space, is $(R/\delta R)^3$. The holographic principle demands that $(R/\delta R)^3 \lesssim (R/l_P)^2$, where $l_P = ct_P \equiv (\hbar G/c^3)^{1/2}$ is the Planck length. This yields

$$\delta R \gtrsim (R l_P^2)^{1/3} = l_P \left(\frac{R}{l_P} \right)^{1/3}. \quad (1)$$

Thus quantum fluctuations from individual bubbles of spacetime inside a distance R add together to produce a (curious) $\sqrt[3]{R}$ -dependence and are much larger than the folklore[2] indicates (viz., $\delta R \gtrsim l_P$).¹ The corresponding metric fluctuation is given by $\delta g_{\mu\nu} \gtrsim (l_P/R)^{2/3}$. Note that even for a macroscopic distance R , the fluctuation δR , though much larger than the Planck scale l_P , is still incredibly small; e.g., for $R = 1$ km, δR is to an atom as an atom is to a human being. Since the holographic principle is deeply rooted in black hole physics, this

¹ Naively, one would assume δR to be given by l_P . Then the number of degrees of freedom would be given by the volume of the region of space R^3 in Planck units. This (wrong) choice of δR violates the holographic principle.

way of deriving spacetime fluctuations is highly suggestive of the deep connection between quantum foam and black hole physics.

From spacetime measurements to spacetime foam

For an alternative means of deriving δR , let us consider a gedanken experiment to measure the distance R between two points. The need for carrying out explicit measurements to determine distances is implicit in general relativity, according to which, coordinates do not have any meaning independent of observations; in fact, a coordinate system is defined only by explicitly carrying out spacetime distance measurements. Following Wigner[3], we can put a clock at one of the points and a mirror at the other. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance. However, the quantum uncertainty in the positions of the clock and the mirror introduces an inaccuracy δR in the distance measurement. Let us concentrate on the clock and denote its mass by m . Wigner argued that if it has a linear spread δR when the light signal leaves the clock, then its position spread grows to $\delta R(2R/c) = \delta R + \hbar R(mc\delta R)^{-1}$ when the light signal returns to the clock, with the minimum at $\delta R = (\hbar R/mc)^{1/2}$. Hence one concludes that

$$\delta R^2 \gtrsim \frac{\hbar R}{mc}. \quad (2)$$

One can supplement this quantum mechanical relation with a limit from general relativity[4]. To see this, let the clock be a light-clock consisting of two parallel mirrors (each of mass $m/2$), a distance l apart, between which bounces a beam of light. For the uncertainty in distance measurement not to be greater than δR , the clock must tick off time fast enough that $l/c \lesssim \delta R/c$. But l , the size of the clock, must be larger than the Schwarzschild radius Gm/c^2 of the mirrors, for otherwise one cannot read the time registered on the clock. From these two requirements, it follows that

$$\delta R \gtrsim \frac{Gm}{c^2}, \quad (3)$$

the product² of which with Eq. (2) yields precisely the expression for spacetime fluctuation δR given by Eq. (1), obtained by using the holographic principle. (Thus one can actually

² Here we can appreciate the importance of taking into account the effects of instruments in this gedanken experiment. Usually when one wants to examine a certain field (say, an electromagnetic field) one uses instruments that are neutral (electromagnetically neutral) and massive for, in that case, the effects of the

argue that the holographic principle has its origin in the quantum fluctuations of spacetime.) A gedanken experiment to measure a time interval T gives an analogous expression:

$$\delta T \gtrsim (T t_P^2)^{1/3}. \quad (4)$$

There are also uncertainties in energy-momentum measurements. Such uncertainties may account for some unexpected features in the high energy cosmic ray and gamma ray spectra.[5]

Interrelationship between spacetime foam and black hole physics

It is interesting that an argument, very similar to that used above to deduce the structure of spacetime foam, can be applied to discuss the precision and the lifetime of a clock.[6] For a clock of mass m , if the smallest time interval that it is capable of resolving is t and its total running time is T , one finds $t^2 \gtrsim \frac{\hbar T}{mc^2}$, [6] the analogue of Eq. (2), and $t \gtrsim \frac{Gm}{c^3}$, the analogue of Eq. (3).³ Now let us apply these two (in-)equalities to a black hole of mass m used as a clock. It is reasonable to use the light travel time across the black hole's horizon as the resolution time of the clock, i.e., $t \sim \frac{Gm}{c^3} \equiv t_{BH}$, then one immediately finds that $T \sim \frac{G^2 m^3}{\hbar c^4} \equiv T_{BH}$, which is just Hawking's black hole lifetime! Thus, if we had not known of black hole evaporation, this remarkable result would have implied that there is a maximum lifetime (of this magnitude) for a black hole. This is another demonstration of the intimate (if, in this case, indirect) relationship between quantum foam and black hole physics.

One can also translate the above clock relations into useful expressions for a simple computer. The fastest possible processing frequency is obviously given by t^{-1} . Thus we identify $\nu = t^{-1}$ as the clock rate of the computer, i.e., the number of operations per bit per unit time. The identification of the number I of bits of information in the memory space of a simple computer is subtler. Since T/t is the maximum number of steps of information

instruments are negligible. But here in our gedanken experiment, the relevant field is the gravitational field. One cannot have a gravitationally neutral yet massive set of instruments because the gravitational charge is equal to the mass according to the principle of equivalence in general relativity. Luckily we can now exploit this equality of the gravitational charge and the inertial mass of the clock to eliminate the dependence on m .

³ One can combine these two expressions to give $T/t^3 \lesssim t_P^{-2} = \frac{c^5}{\hbar G}$, which relates clock precision to its lifetime.[6] (Note that this new expression is just Eq. (4) with t playing the role of δT .)

processing, we make the identification $I = T/t$.⁴ Now imagine that we can form a black hole (of mass m) whose initial conditions encode certain information to be processed.⁵ Then the memory space of the black hole computer has $I = T_{BH}/t_{BH} \sim (m/m_P)^2$, where $m_P \equiv (\hbar c/G)^{1/2}$ is the Planck mass. This gives the number of bits I as the event horizon area in Planck units, as expected from the identification[1] of a black hole entropy! Furthermore, the number of operations per unit time for a black hole computer is given by $I\nu \sim mc^2/\hbar$, in agreement with Lloyd's results[7] for the ultimate physical limits to computation. All these results indicate the conceptual interconnections of the physics underlying simple clocks, simple computers, black holes, and spacetime foam.

Interferometers as detectors of spacetime foam

Now we come to an important question: how do we detect quantum foam, i.e., how do we check Eq. (1) and Eq. (4) experimentally? It has been suggested that modern gravitational-wave interferometers can potentially provide a way, because the intrinsic foaminess of spacetime gives another source of noise in the gravitational-wave interferometers that can be highly constrained.[8] Here we propose to use a smaller and simpler experimental setup; and following a similar analysis by I. Percival[9], we optimistically suggest that laser-based atom interferometry experiments may be precise enough in the not-too-distant future to detect spacetime fluctuations on the scales of quantum gravity at the level given by Eq. (1) and Eq. (4). In a laser-based atom interferometer, an atomic beam is split by laser beams into two coherent wave packets which are kept apart before being recombined by laser beams. The phase change of each wave packet is proportional to the proper time along its path, and so the resulting interference pattern depends on the time difference between the two paths. In the absence of spacetime fluctuations, the phase change η over a time interval T is given by $\eta(T) = \Omega T$, where $\Omega \equiv mc^2/\hbar$ is the quantum angular frequency associated

⁴ Using the clock relation in the preceding footnote and the identifications of ν and I in terms of t and T , one gets $I\nu^2 \lesssim \frac{c^5}{\hbar G}$. This expression links together our concepts of information, gravity, and quantum uncertainty.[6]

⁵ It is possible, in principle, to program black holes to do computations in such a way that the results of the computation can be read out of the fluctuations in the apparently thermal Hawking radiation, if black holes indeed evolve in a unitary fashion as we believe.[7] For a black hole computer, the inequality $I\nu^2 \lesssim \frac{c^5}{\hbar G}$ is saturated; thus one can even claim that black holes, once they are programmed to do computations, are the ultimate computers.

with the mass m of the atom. Due to spacetime fluctuations (Eq. (4)), there is an additional fluctuating phase $\delta\eta$ given by

$$\delta\eta \sim \frac{(Tt_P^2)^{1/3}}{T}\eta = (Tt_P^2)^{1/3}\Omega. \quad (5)$$

For example, in 1992, Chu and Kasevich at Stanford University built an atom interferometer which used sodium atoms ($m \sim 4.5 \times 10^{-26}$ kg), and the two wave packets were kept apart for 0.2 sec.[10] For that experiment, one finds that $\eta(T) \sim 7 \times 10^{24}$ radians and $\delta\eta \sim 3 \times 10^{-4}$ radians. Thus one needs a precision of about 1 part in 10^{29} to look for spacetime foam (through suppression of the interference pattern), compared with the precision of 1 part in 10^{26} that was then achieved. In other words, one needs a (mere) thousandfold improvement in noise sensitivity to detect spacetime fluctuations. Though the above argument, a variant of the one given in Ref. [9], is necessarily short and perhaps too simplistic and overtly optimistic, hopefully the conclusion is not too far off the mark.

In summary, we have combined the general principles of quantum mechanics and general relativity to address the problem of quantum fluctuations of spacetime. A simple application of the holographic principle has shown that spacetime undergoes much larger quantum fluctuations than one may expect. This result is confirmed by gedanken experiments for spacetime measurements. We believe that the Planck scale, so far only a hypothetical extreme regime, will eventually become a realm that can be approached and measured, for instance by interferometry techniques. In this essay, we have also highlighted the interconnection between spacetime foam and black hole physics. Hence, if future experiments show that spacetime fluctuates at a level smaller than our prediction $\delta R \sim \sqrt[3]{Rl_P^2}$, we will know that our current understanding of semiclassical gravity and black hole physics may need a considerable revision. We hope that these arguments are sufficiently convincing to encourage a determined experimental quest to detect quantum foam, the very fabric of spacetime, for, as Faraday wrote:

*Nothing is too wonderful to be true, if it be consistent with the laws of nature,
and in such things as these, experiment is the best test of such consistency.*

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